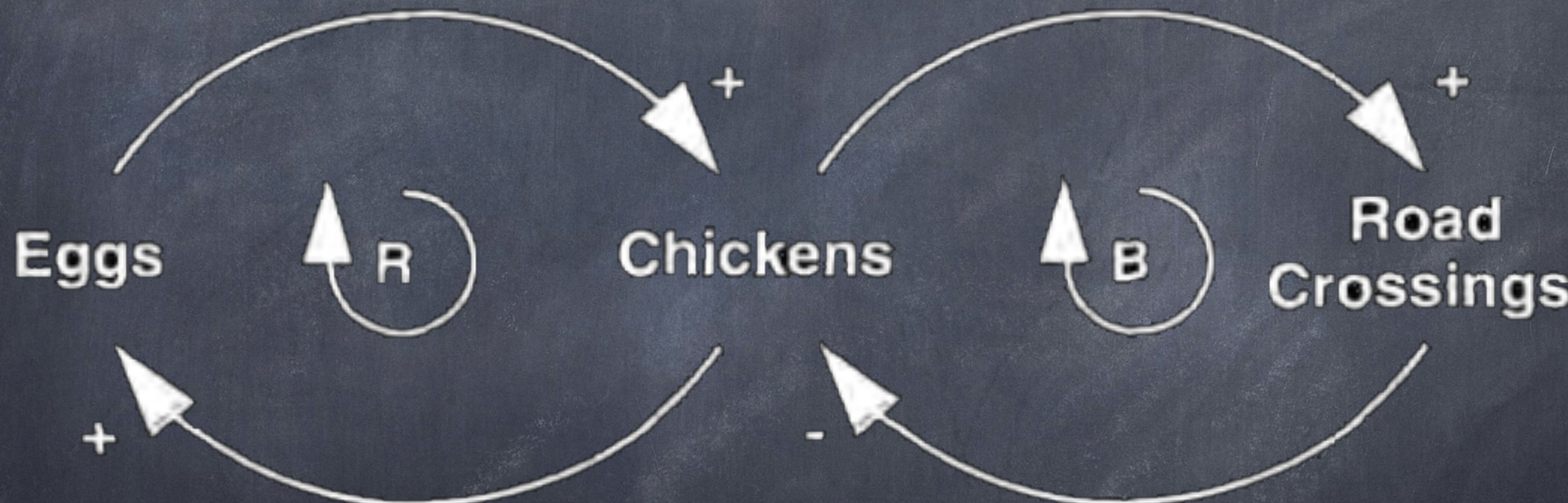
System Dynamics Modeling at MIT & Around the World



from Tim Havel to
the MIT Applied Category Theory Seminar

What is SD Modeling ?

- From a strictly mathematical viewpoint, it's "just" non-linear ODE's (with feedback and often external drives aka time-dependence)
- The "art" is how to apply such models to the real world, including but not limited to
  - Sociology and behavioral economics
  - Psychology of learning and decision making
  - Epidemiology and "viral" collective behavior
  - Ecosystem dynamics and collapse

Where Did It Come From?

- Started in the early 60's by Jay Forrester modeling business processes at the MIT Sloan School of Management (where I studied it)
- Achieved considerable notoriety when used to model the collapse of civilization in the 1972 best seller "The Limits to Growth"
- Now globally coordinated by the SD Society, with chapters in all major countries and annual meetings (this year in Reykjavík)

Why Might any of You Care?

- Its practitioners use similar diagrammatic languagesto communicate with each other and setup their models for computer simulation
- These diagrams, simulations and graphical outputs makes it possible for people with little math background to qualitatively understand ODE's that otherwise defy rigorous analysis
- There's money in it (not a lot, but some big companies & management consulting firms us it)

What It's Really About

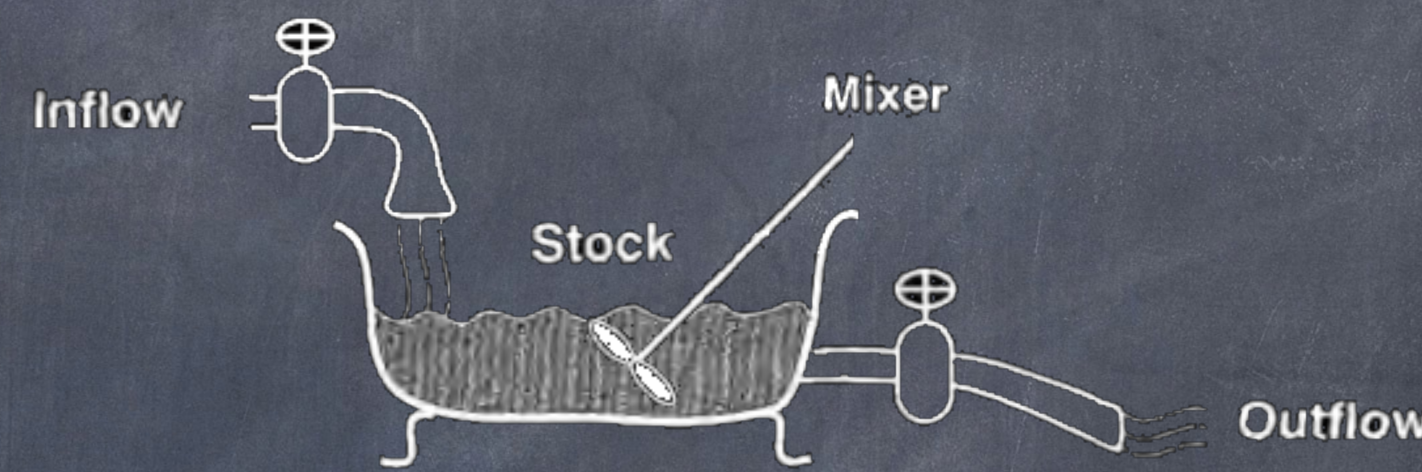
While bounded rationality affects all decision contexts, it is particularly acute in dynamic systems. Indeed, experimental studies show that people do quite poorly in systems with even modest levels of dynamic complexity. These studies led me to suggest that the observed dysfunction in dynamically complex settings arises from misperceptions of feedback. The mental models people use to guide their decisions are dynamically deficient. As discussed above, people generally adopt an event-based, open-loop view of causality, ignore feedback processes, fail to appreciate time delays between action and response and in the reporting of information, do not understand stocks and flows and are insensitive to nonlinearities that may alter the strengths of different feedback loops as a system evolves.

John Sterman, Business Dynamics, page 27

Integrals as Bathtubs

Otherwise known as "Calculus for Tots"

Hydraulic Metaphor:



Stock and Flow Diagram:

Integral Equation:

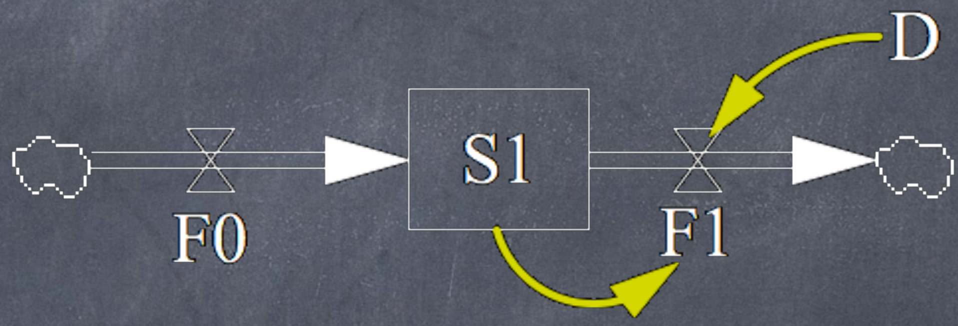
$$\text{Stock}(t) = \int_{t_0}^{t} [\text{Inflow}(s) - \text{Outflow}(s)]ds + \text{Stock}(t_0)$$

Differential Equation:

$$d(\text{Stock})/dt = \text{Net Change in Stock} = \text{Inflow}(t) - \text{Outflow}(t)$$

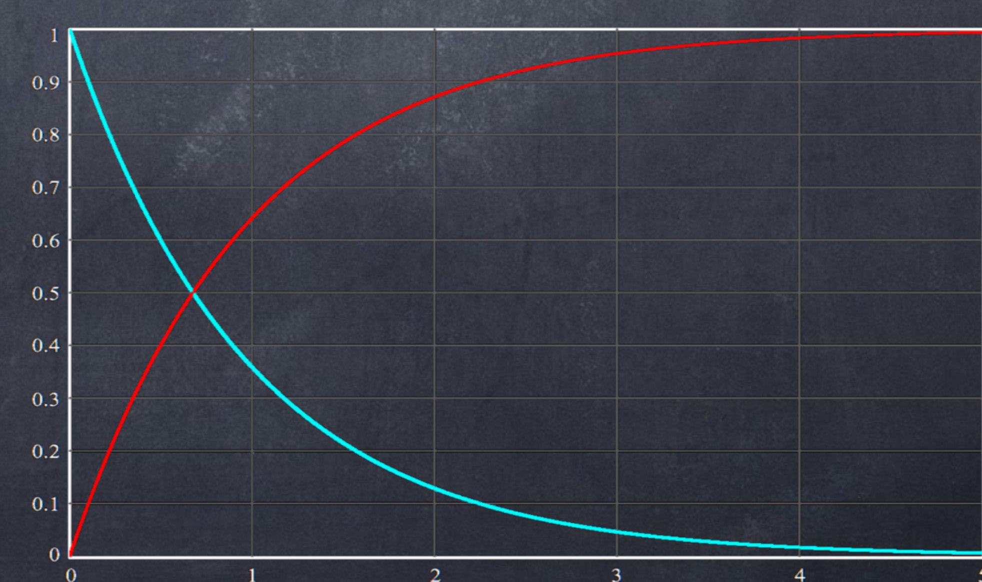
1st Order Delays instead of Derivatives

- In the simplest case:
  - if $F_0 = 0$, decay is exponential from C_1
  - if $C_1 = 0$ & F_0 is a constant, grows to plateau at $D \times F_0$
- Subsumes 1st order linear non-autonomous ODE



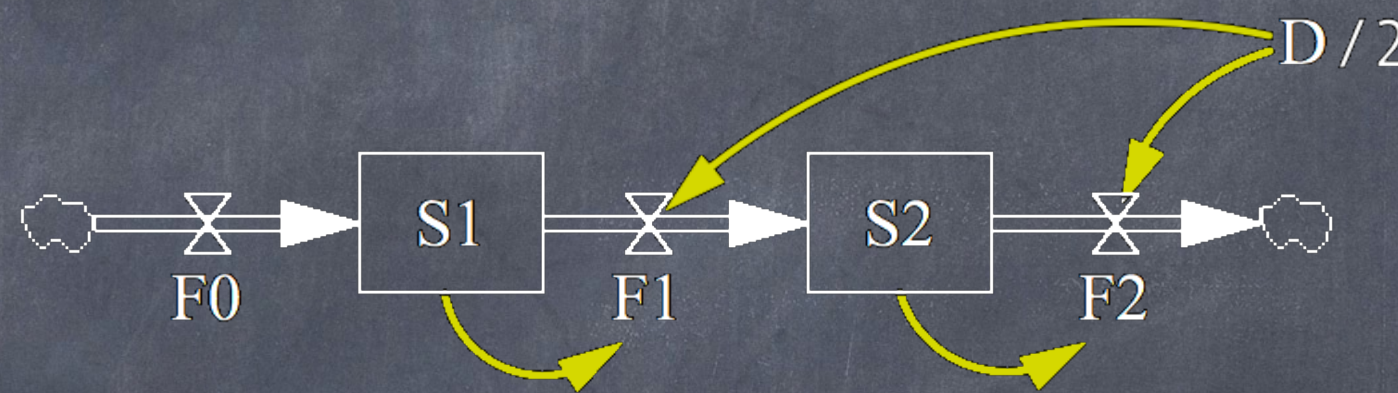
$$\frac{dS_1}{dt} = F_0(t) - F_1(t) = F_0(t) - \frac{S_1(t)}{D}$$

$$S_1(t) = e^{-t/D}\left(\int_0^t ds\, e^{s/D} F_0(s) + C_1\right)$$



2nd & Higher Order Delays

- Formal solution to a 2nd order delay D = D/2 + D/2

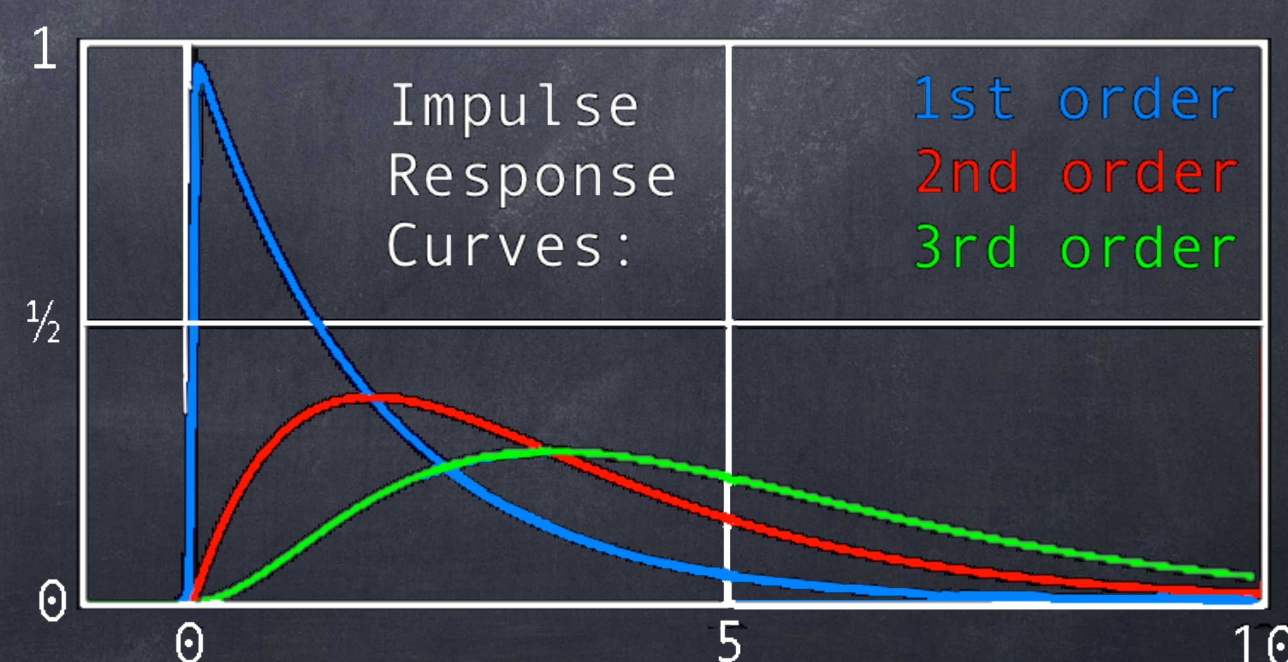


$$S_2(t) = e^{-2t/D}\left(\int_0^t ds\, e^{2s/D}\frac{2\,S_1(s)}{D} + \bar{S}_2\right)$$

$$= e^{-2t/D}\left(\int_0^t ds\left(\frac{2}{D}\int_0^s dr\, e^{2r/D}F_0(r) + \bar{S}_1\right) + \bar{S}_2\right)$$

- Impulse response of 1st, 2nd & 3rd order delays

- nth order goes to "pipeline" delay as $n \to \infty$

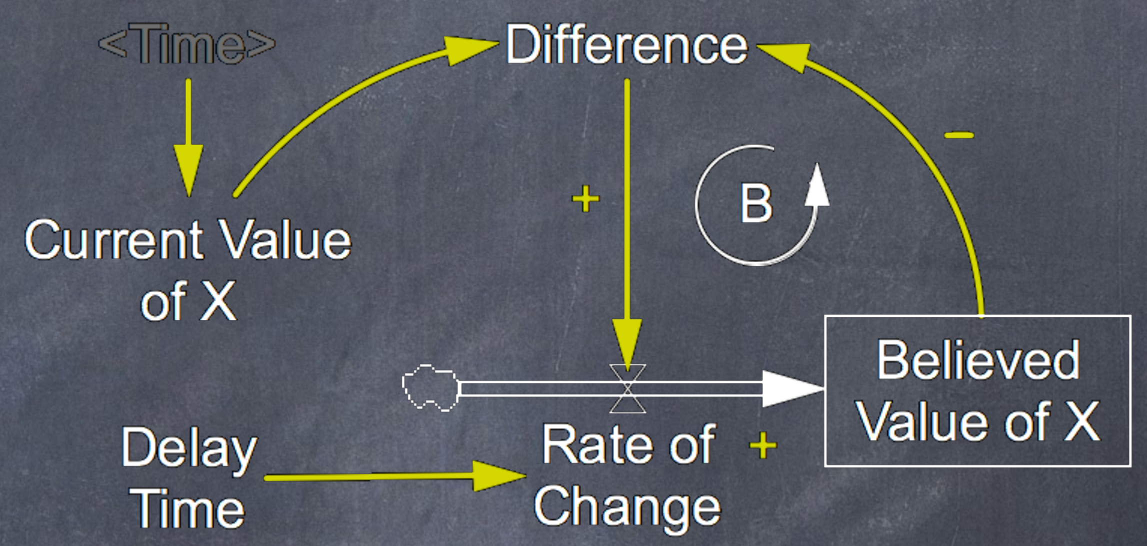


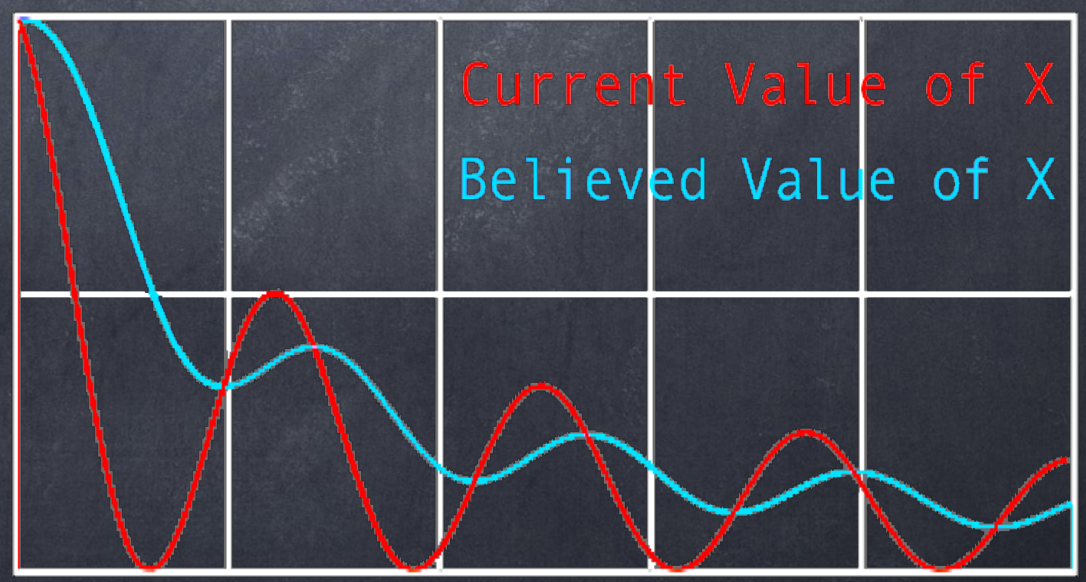
Material vs Information Delays

- What we have seen so far are "material" delays: the output of one stock is the input of the next, with conservation of flow
- System dynamics also makes extensive use of "information" delays to model a person/group:
  - learning a real valued parameter
  - forgetting a real valued parameter
  - updating their belief of the parameter's value

Generic Information Delay: The "P" in a "PID" Controller

- Sequential info delays can simulate material delays of an order (but shouldn't!)
- Flow in direction of arrow may become negative, as may the corresponding stocks
- 1st order I.D.'s most common, & correspond to exponential smoothing of input time series

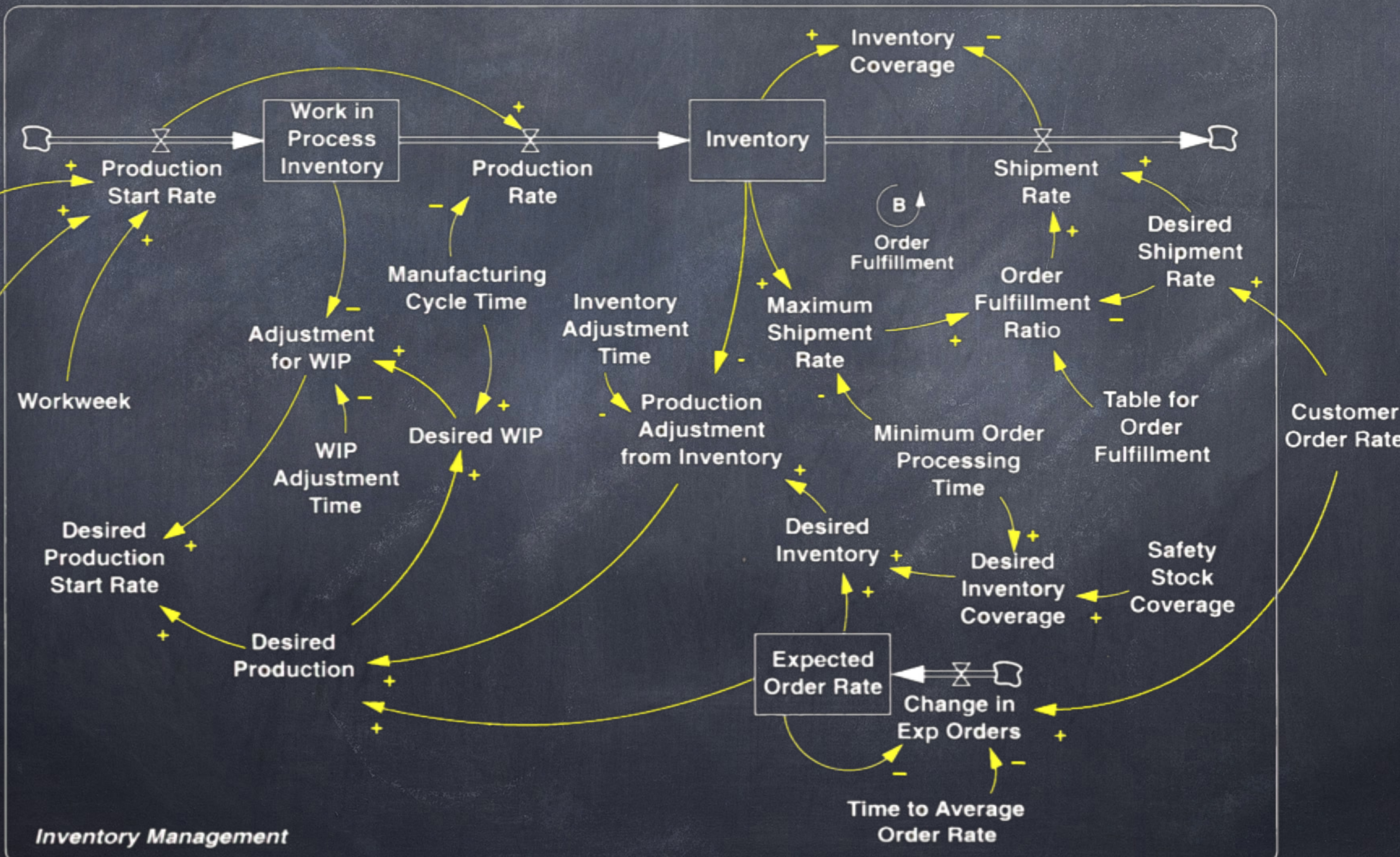




The syntax of System Dynamics Diagrams is defined by 2 simple rules:

1. The subgraph induced by all non-stock variables is a D.A.G.
2. All loops must pass through at least one stock variable

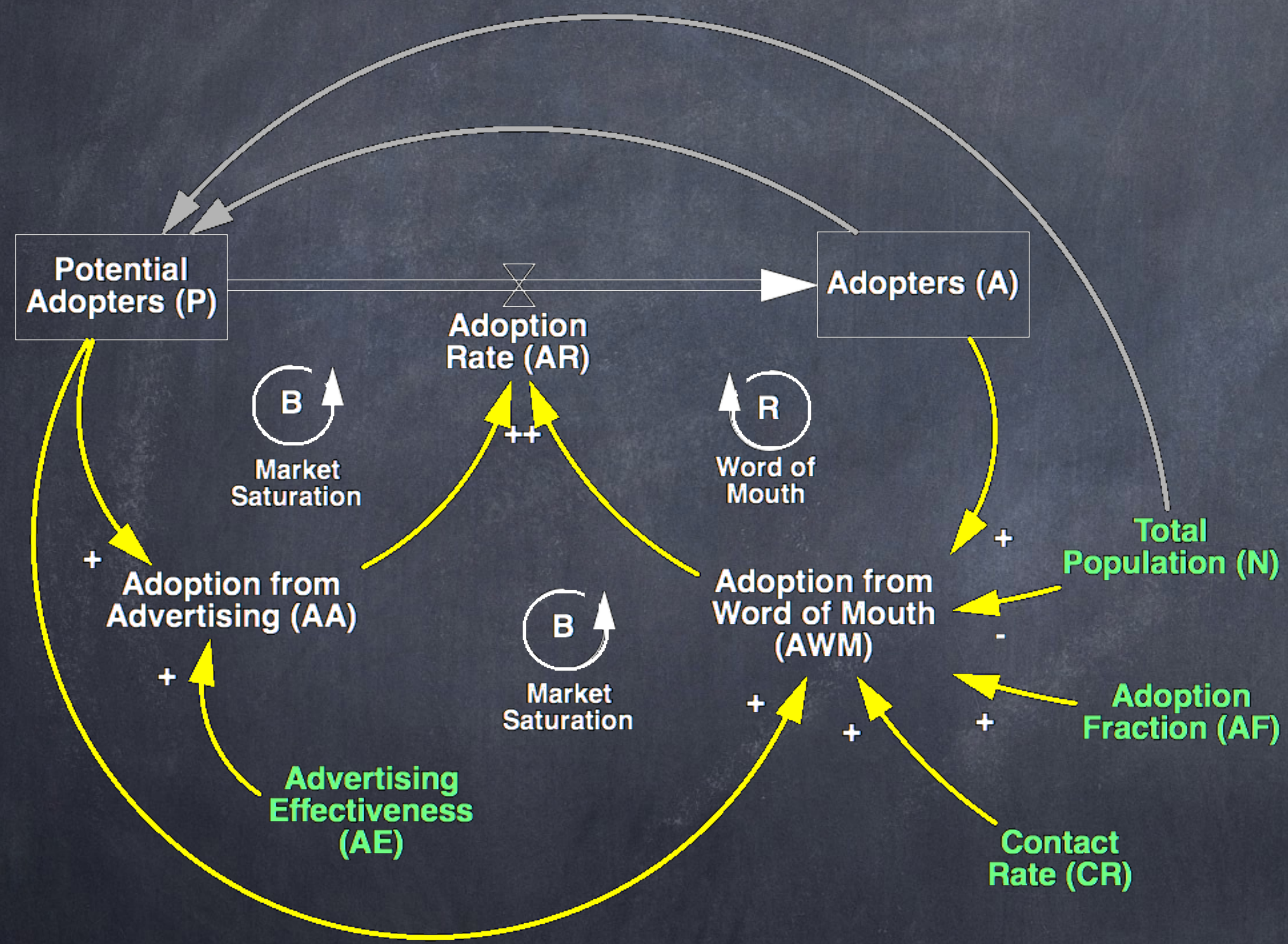
(other loops violate causality, with no delay between cause & effect)

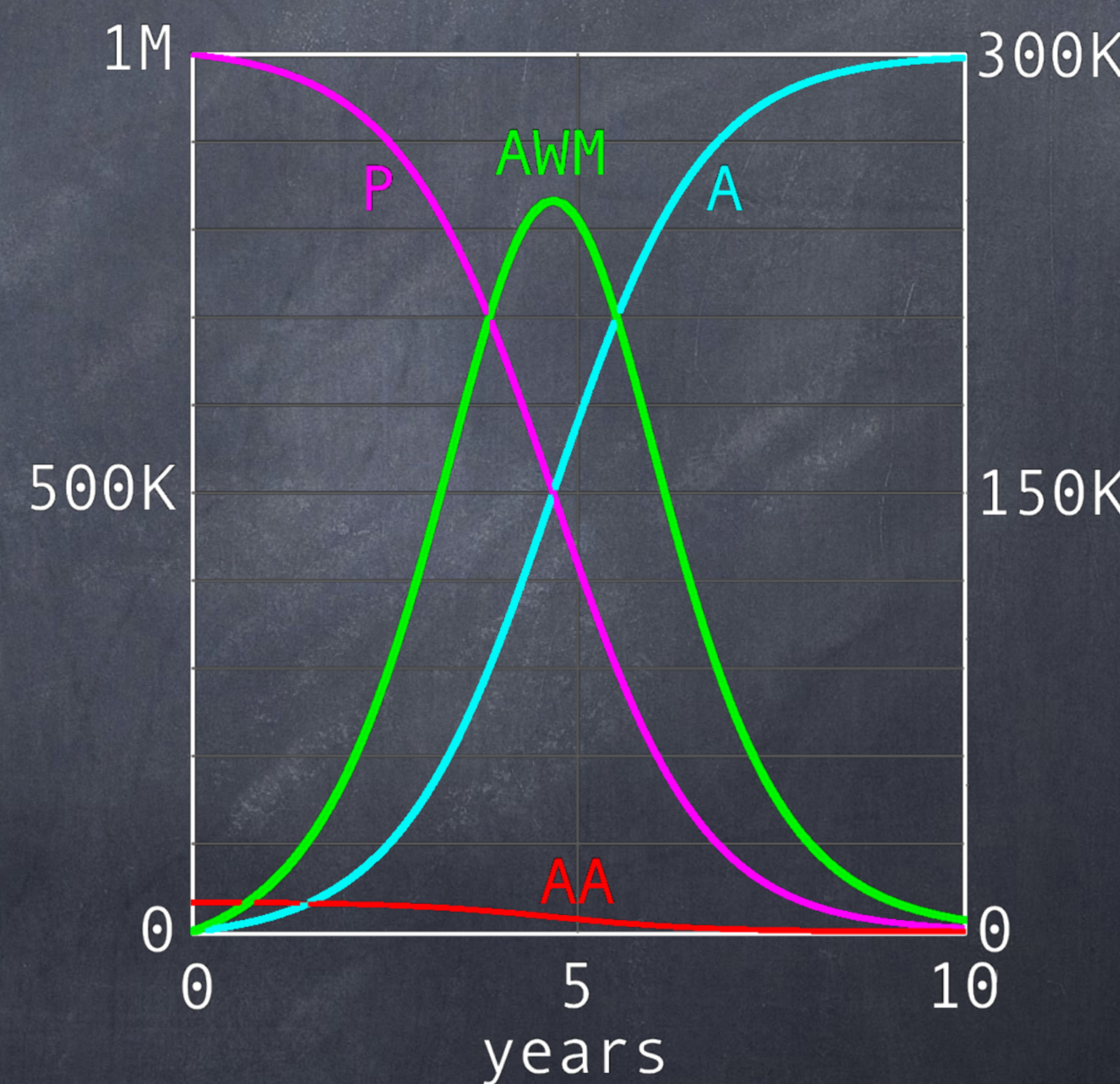


Some Remarks on the Nature of Problems SD People Encounter

- Although they may oscillate along the way, all the dynamical systems I've seen come out of SD modeling tend towards a steady state
- Feedback loops that can change from positive to negative are seen as undesirably complex; disaggregate into two competing loops instead
- Likewise, stocks (even info) are usually non-negative, and exceptions tend to have obvious explanations

Example #1: The Bass Diffusion Model

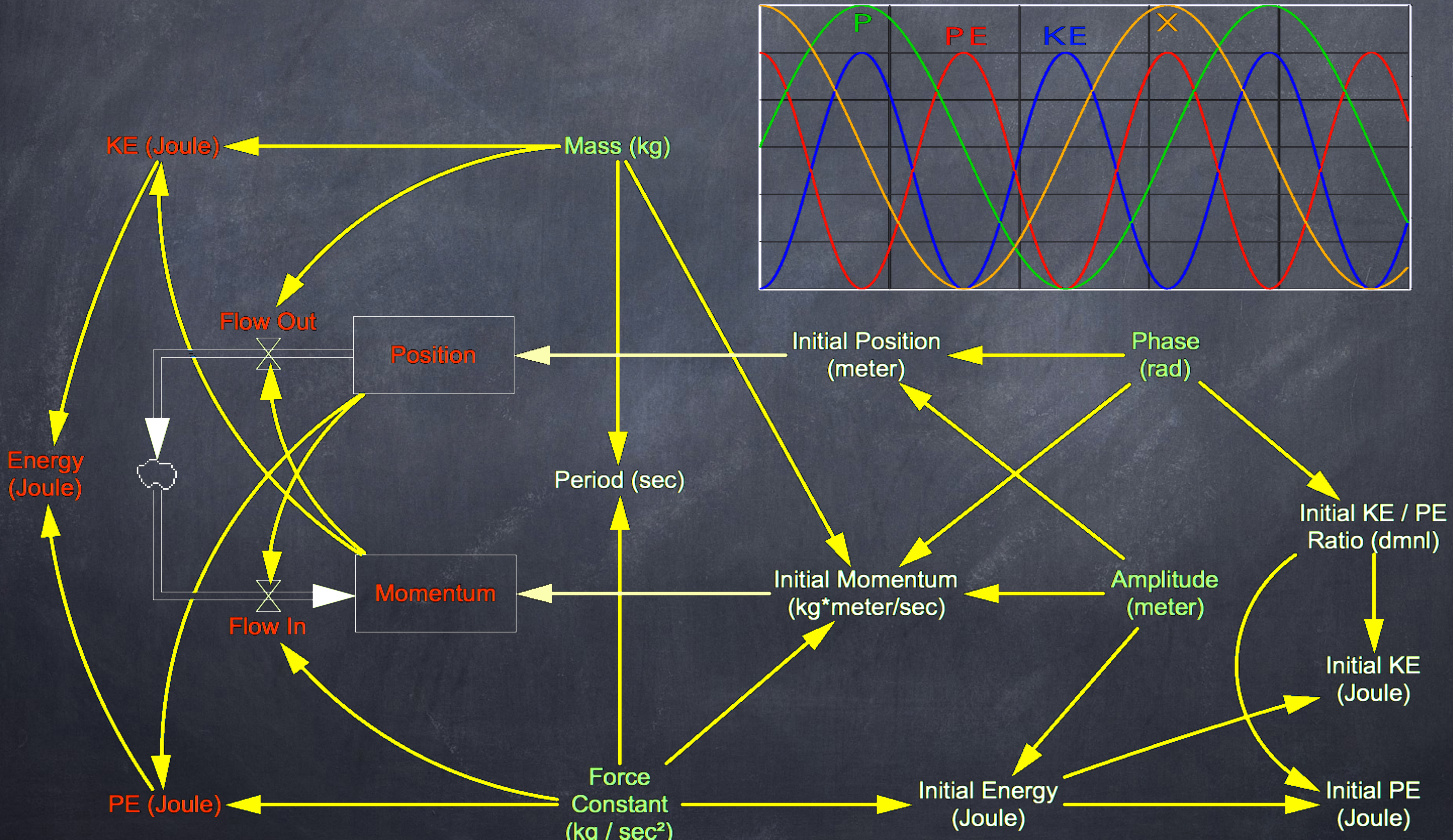




A = INT(AR, A°)   AR = AA + AWM   AWF = CR x AF x A x P / N

P = INT(–AR, N – A°)   AA = AE x P   AE = 0.01, AF = 0.01, CR = 100, A° = 0

Example #2: The Harmonic Oscillator



Example #3: The Dimensionless Brusselator

$$A \longrightarrow X$$
$$B + X \longrightarrow D + Y$$
$$2X + Y \longrightarrow 3X$$
$$X \longrightarrow E$$

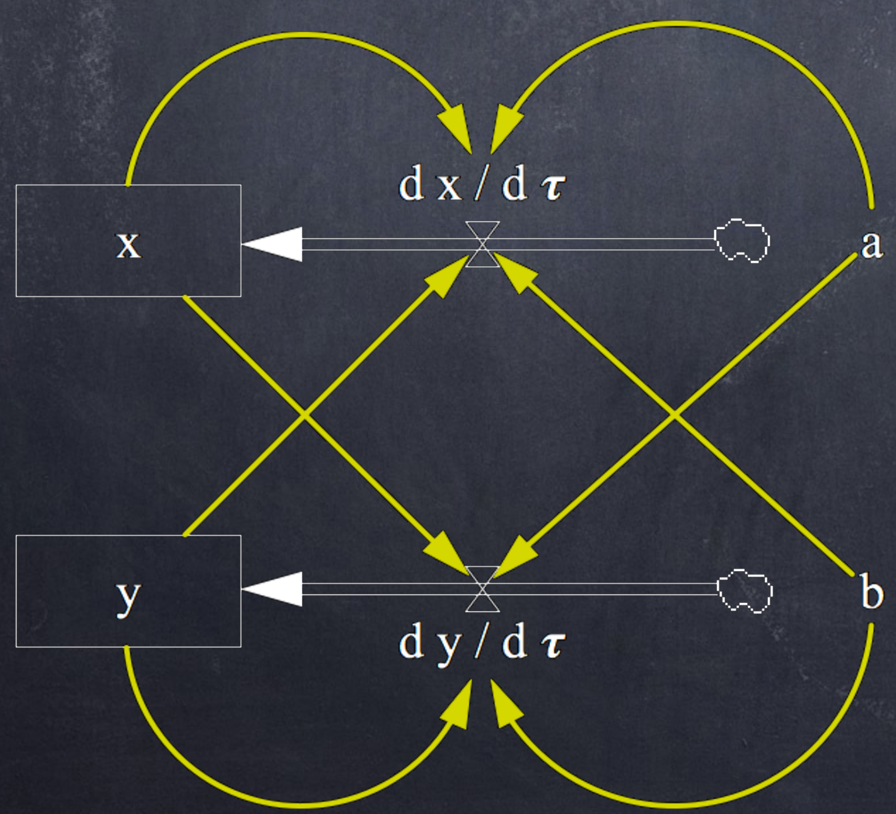
$$\frac{d[X]}{dt} = k_1[A] - k_2[B][X] + k_3[X]^2[Y] - k_4[X]$$

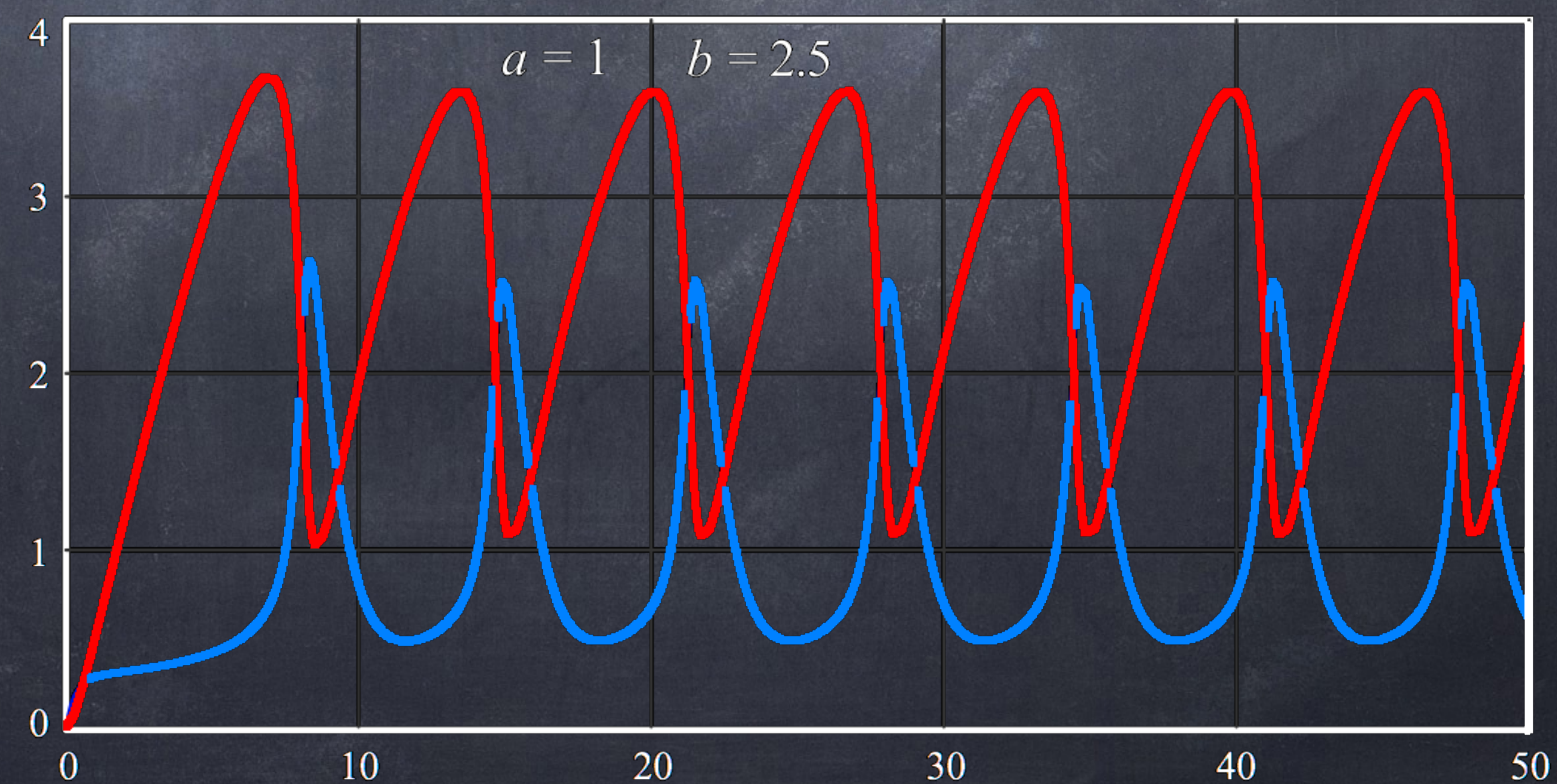
$$\frac{d[Y]}{dt} = k_2[B][X] - k_3[X]^2[Y]$$

$$[X]_{ss} = \frac{k_1[A]}{k_4}, \quad [Y]_{ss} = \frac{k_2 k_4 [B]}{k_1 k_3 [A]}$$

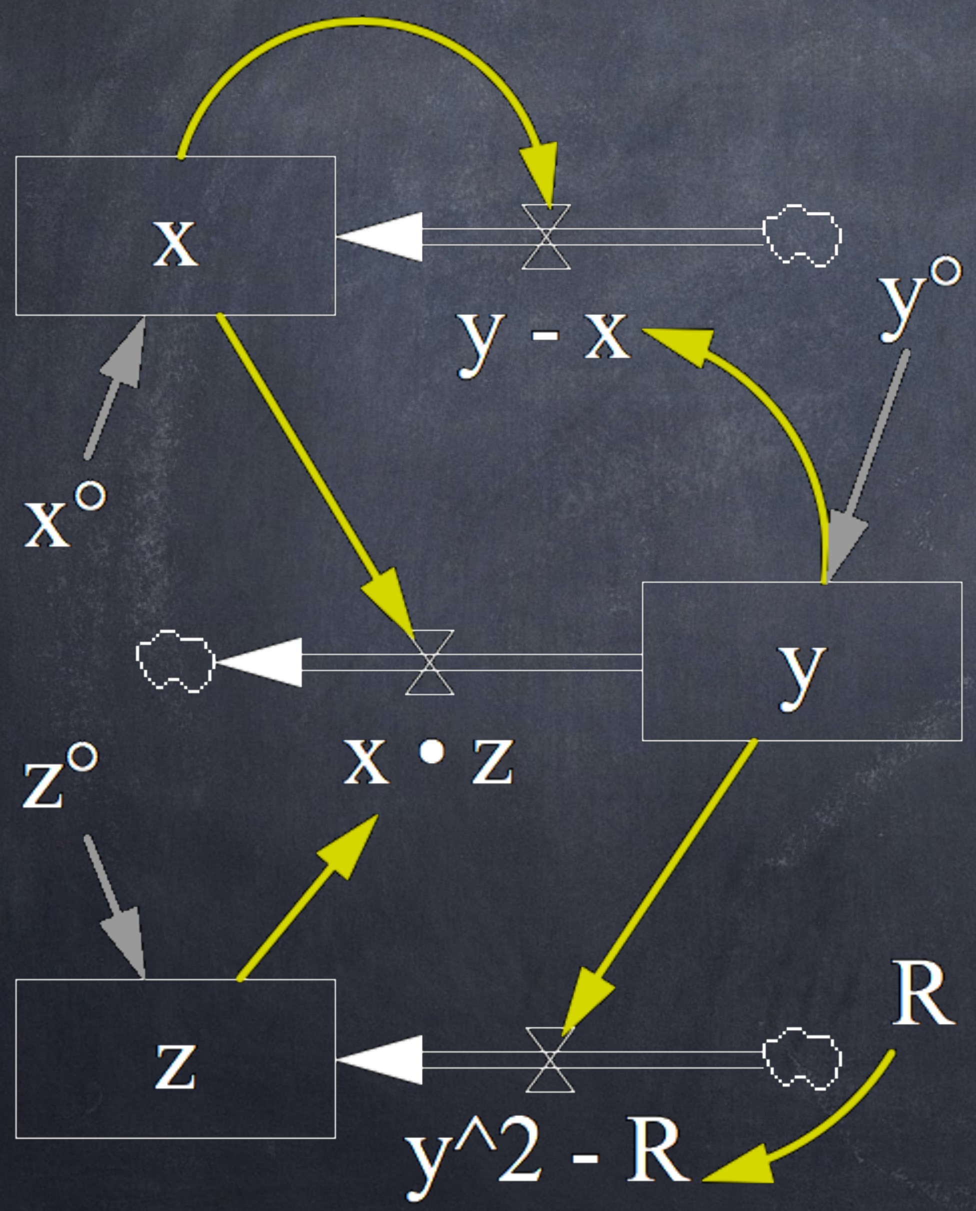
$$\frac{dx}{d\tau} = 1 - (1+b)x + ax^2 y$$

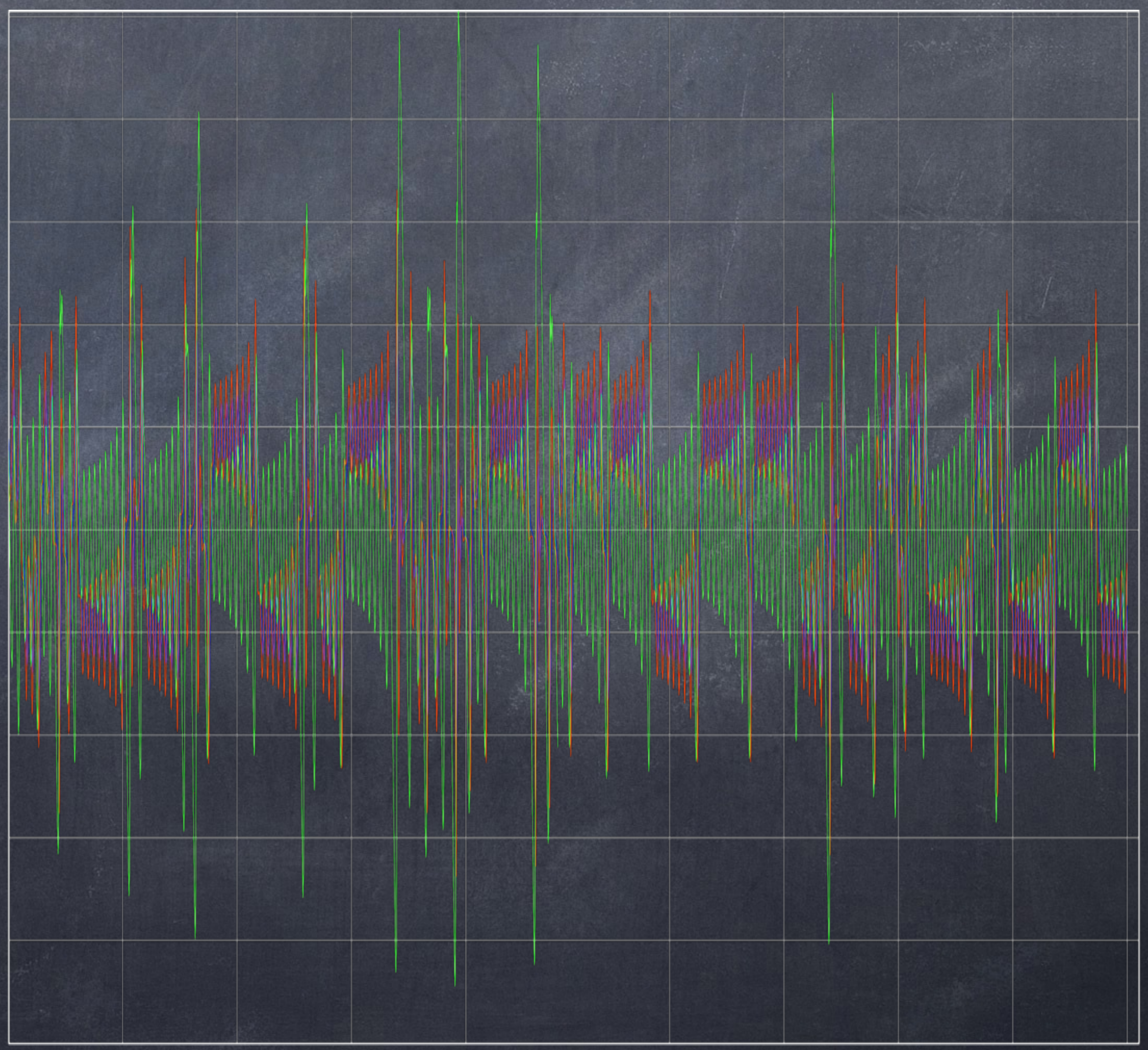
$$\frac{dy}{d\tau} = bx - ax^2 y$$





Example #4: A Simplified Chaotic Lorentz Model





Example #5: The Limits to Growth World Model

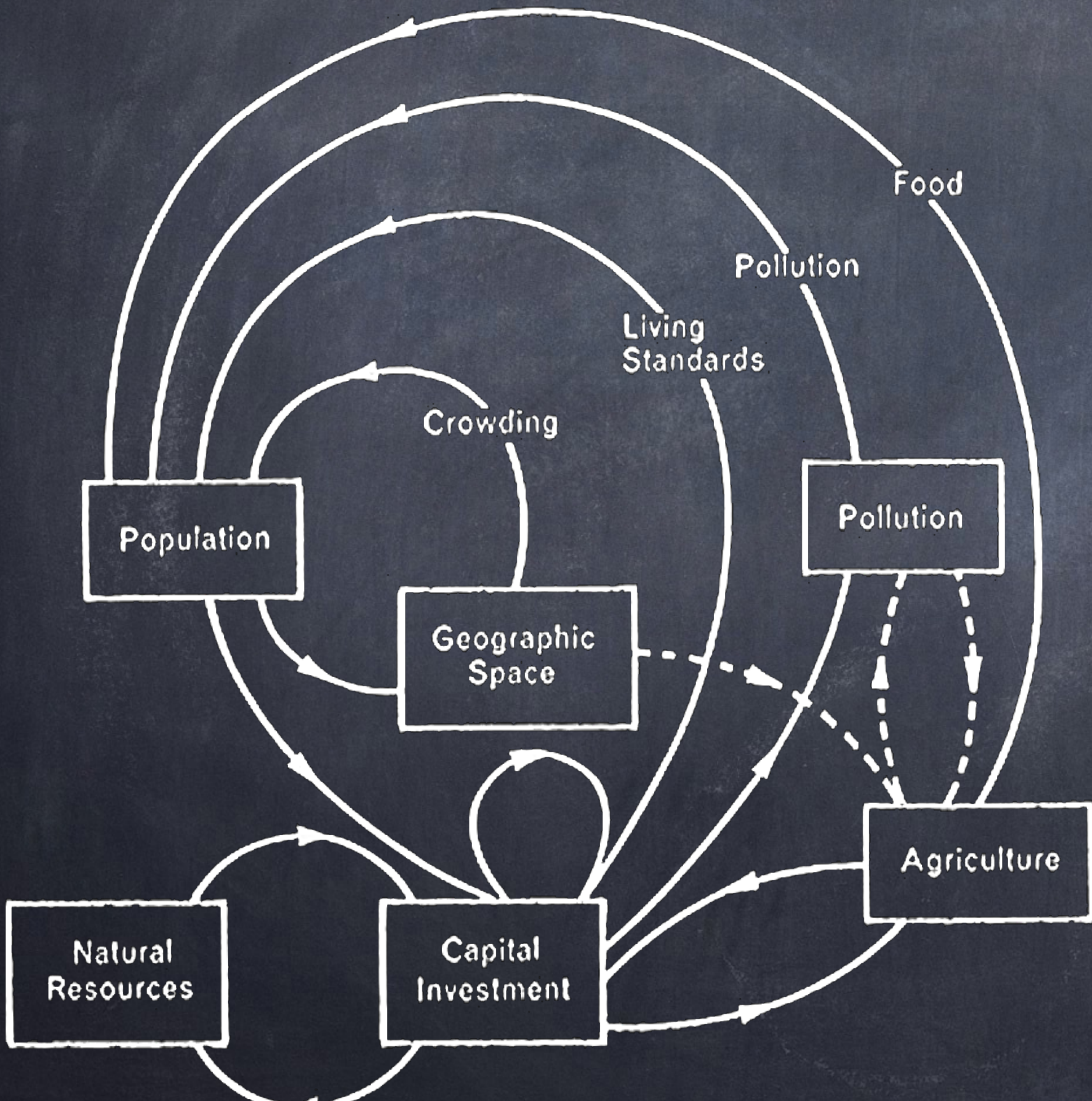
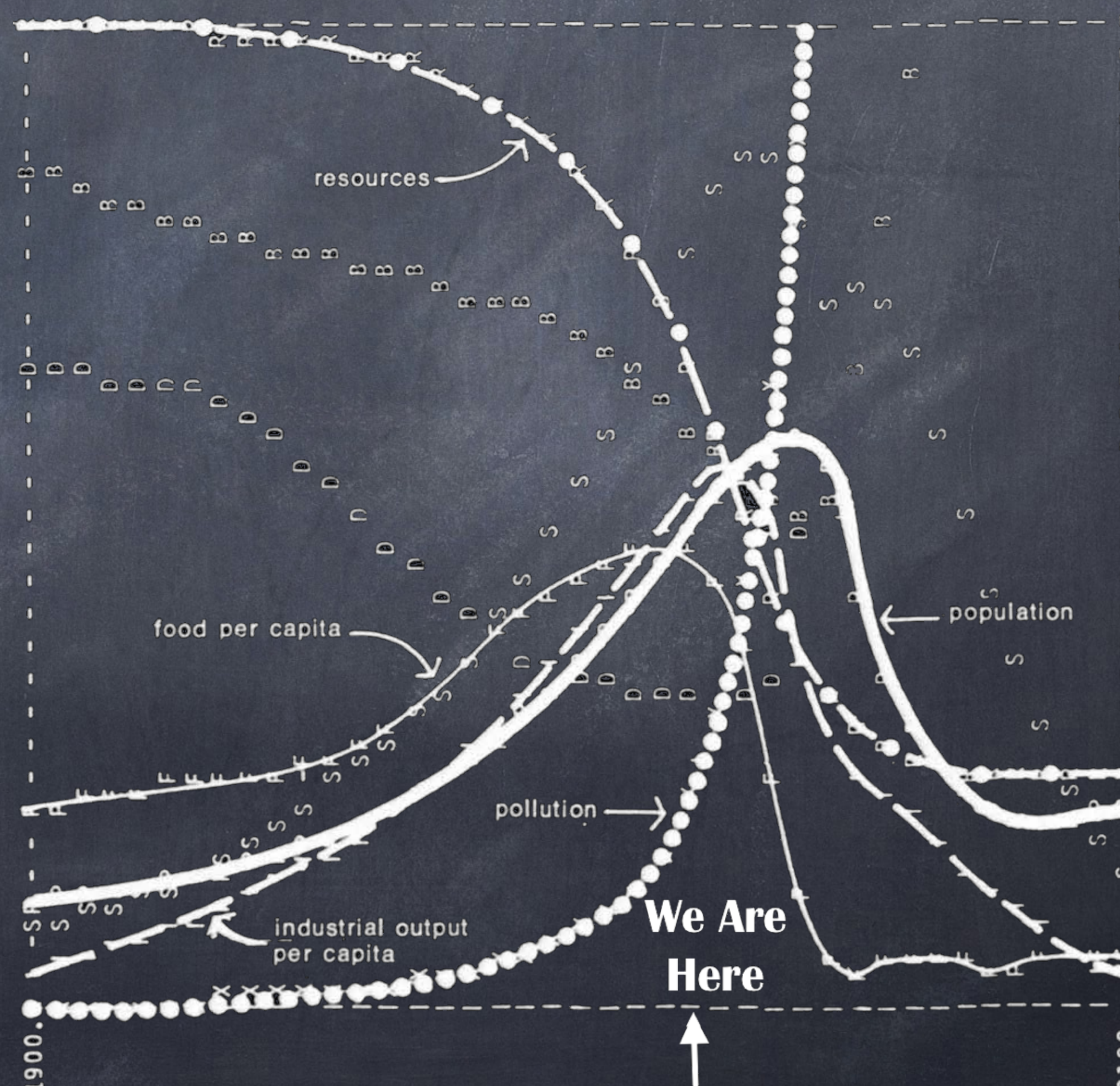
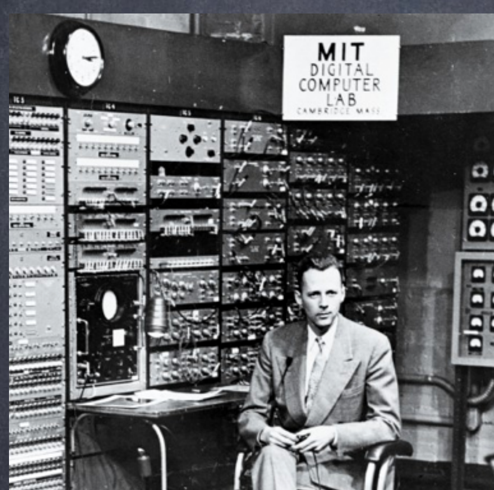


Figure 36 WORLD MODEL WITH NATURAL RESOURCE RESERVES DOUBLED



Additional Resources

www.systemdynamics.org:
history, models, membership, annual conference



MIT System Dynamics Group Literature Collection selected by Jay W. Forrester

The MIT System Dynamics Group Literature Collection is a powerful tool that will advance your knowledge of the important history of the field at MIT and help build a foundation for understanding future growth. The collection, based on the famous D-memos, spans nearly fifty years of work in system dynamics and reveals a rich historical point of view.

vensim.com/resources:
list of wikis, blogs, books, etc.

en.wikipedia.org/wiki/System_dynamics: